

Exercise 1.48

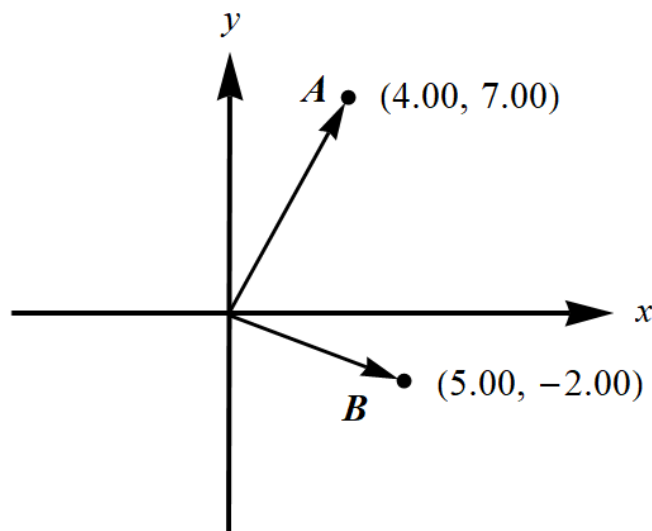
Find the vector product $\vec{A} \times \vec{B}$ (expressed in unit vectors) of the two vectors given in Exercise 1.42. What is the magnitude of the vector product?

Solution

The two vectors from Exercise 1.42 are

$$\begin{aligned}\mathbf{A} &= 4.00\hat{i} + 7.00\hat{j} \\ \mathbf{B} &= 5.00\hat{i} - 2.00\hat{j}.\end{aligned}$$

They're illustrated in the xy -plane below.



The vector (cross) product is obtained by evaluating a 3×3 determinant.

$$\begin{aligned}\mathbf{A} \times \mathbf{B} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \\ &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4.00 & 7.00 & 0 \\ 5.00 & -2.00 & 0 \end{vmatrix} \\ &= \begin{vmatrix} 7.00 & 0 \\ -2.00 & 0 \end{vmatrix} \hat{i} - \begin{vmatrix} 4.00 & 0 \\ 5.00 & 0 \end{vmatrix} \hat{j} + \begin{vmatrix} 4.00 & 7.00 \\ 5.00 & -2.00 \end{vmatrix} \hat{k} \\ &= [(7.00)(0) - (0)(-2.00)]\hat{i} - [(4.00)(0) - (0)(5.00)]\hat{j} + [(4.00)(-2.00) - (7.00)(5.00)]\hat{k} \\ &= 0\hat{i} - 0\hat{j} - 43.00\hat{k} \\ &= -43.00\hat{k}\end{aligned}$$

Its magnitude is

$$|\mathbf{A} \times \mathbf{B}| = \sqrt{(-43.00)^2} = 43.00.$$